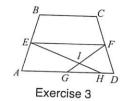
3. Given: \overline{EF} is the median of trapezoid ABCD

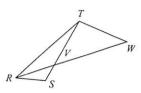
Prove: $EI \times GH = IH \times FF$



4. Given: V is a point on \overline{ST} such that \overline{RVW} bisects $\angle SRT$

 $\overline{TW} \cong \overline{TV}$.

Prove: $RW \times SV = RV \times TW$



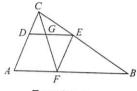
Exercise 4

5. Given: $\triangle ABC$ with \overline{CDA} , \overline{CEB} . $\overline{AFB}, \overline{DE} \parallel \overline{AB}, \overline{EF} \parallel \overline{AC}.$

 \overline{CF} intersects \overline{DE} at G.

Prove: a. $\triangle CAF \sim \triangle FEG$

b. $DG \times GF = EG \times GC$



Exercise 5

6. Rosalie thinks she discovered a new theorem. She claims that "The product of the lengths of the legs of a right triangle is always equal to the product of the lengths of the hypotenuse and the altitude drawn to the hypotenuse." Prove or disprove Rosalie's theorem.

CHAPTER 6

RIGHT TRIANGLES AND TRIGONOMETRY

PROPORTIONS IN A RIGHT TRIANGLE



Drawing an altitude to the hypotenuse of a right triangle creates pairs of similar right triangles from which useful proportions can be derived.

Wean Proportional

If the means of a proportion are the same, as in $\frac{8}{4} = \frac{4}{2}$, then either mean is

med the mean proportional or geometric mean between the other two bers. Thus, 4 is the mean proportional between 8 and 2. Equivalently, 4 the geometric mean of 8 and 2.

Example 1

Find the mean proportional between:

 $\frac{1}{2}$ 2 and 50 b. 6a and $8a^3$

Solution: Let x represent the mean proportional between the given pair of numbers.

a.
$$\frac{4}{x} = \frac{x}{25}$$

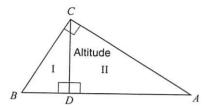
 $x \cdot x = 4 \cdot 25$
 $x^2 = 100$
 $x = \sqrt{100} = 10$.

b. $\frac{6a}{x} = \frac{x}{8a^3}$
 $x^2 = 48a^4$
 $x = \sqrt{48} \cdot \sqrt{a^4}$
 $= \sqrt{16} \cdot \sqrt{3} \cdot \frac{1}{3}$

Right Triangle Proportions

The altitude drawn to the hypotenuse of a right triangle separates it into other right triangles that are similar to each other and to the original right angle. In the accompanying figure of right triangle ABC,

- $\triangle I \sim \triangle ABC$ since the two right triangles have $\angle B$ in common.
- \triangle II ~ $\triangle ABC$ as the two right triangles have $\angle A$ in common.
- △I ~ △II because two triangles similar to the same triangle are some to each other (Transitive Property of Similarity).



Theorem: Altitude-Hypotenuse Theorem

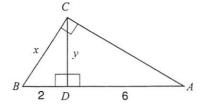
If the altitude is drawn to the hypotenuse of a right triangle, then the two right triangles that are formed are similar to each other and to the original right triangle.

Two important corollaries of this theorem tell how the segments formed the hypotenuse are related to the legs and to the altitude on the hypotenuse

Corollary	Proportions	Figure
Corollary 1: The altitude to the hypotenuse of a right triangle divides the hypotenuse so that either leg is the mean proportional between the hypotenuse and the segment of the hypotenuse adjacent to that leg.	$\frac{x}{a} = \frac{a}{c}$ and $\frac{y}{b} = \frac{b}{c}$	
Corollary 2: The altitude to the hypotenuse of a right triangle is the mean proportional between the two segments along the hypotenuse.	$\frac{x}{h} = \frac{h}{y}$	

mple 2

The accompanying diagram of right margle ABC, \overline{CD} is the altitude to accompanying diagram of right to the accompanying diagram of right \overline{AB} . Find the values of



Solution: The length of hypotenuse \overline{AB} is 2 + 6 = 8.

 \square Corollary 1 to find x:

$$\frac{BD}{x} = \frac{x}{AB}$$

$$\frac{2}{x} = \frac{x}{8}$$

$$x^2 = 16$$

$$x = \sqrt{16} = 4$$

Use Corollary 2 to find *y*:

$$\frac{BD}{y} = \frac{y}{AD}$$

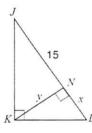
$$\frac{2}{x} = \frac{x}{6}$$

$$x^2 = 12$$

$$x = \sqrt{4 \cdot 3} = 2\sqrt{3}$$

Example 3

the accompanying diagram of right mangle JKL, \overline{KN} is the altitude to potenuse \overline{LJ} . Find the values of and y.



Solution: The length of hypotenuse \overline{LJ} is represented by x + 15.

 \subseteq Corollary 1 to find x:

$$\frac{LN}{10} = \frac{10}{LJ}$$

$$\frac{x}{10} = \frac{10}{x+1}$$

$$x(x+15) = 100$$

$$x^2 + 15x - 100 = 0$$

$$(x-5)(x+20) = 0$$

$$x-5 = 0 \quad \text{or} \quad x+20 = 0$$

Use Corollary 2 to find *y*:

$$\frac{LN}{y} = \frac{y}{NJ}$$

$$\frac{5}{x} = \frac{x}{15}$$

$$x^2 = 75$$

$$x = \sqrt{25 \cdot 3}$$

$$= 5\sqrt{3}$$

must be positive

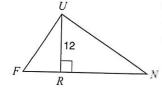
x = 5 or x = -20 \leftarrow Reject since x

Check Your Understanding of Section 6.1

A. Multiple Choice

- 1. If the length of the altitude drawn to the hypotenuse of a right triange # 10 inches, the number of inches in the lengths of the segments of the hypotenuse may be
 - (1) 5 and 20 (2) 2 and 5
- (3) 3 and 7
- (4) 50 and 50

2. In the accompanying diagram, $\triangle FUN$ is a right triangle, \overline{UR} is the altitude to hypotenuse \overline{FN} , UR = 12, and the lengths of \overline{FR} and \overline{FN} are in the ratio 1:10. What is the length of \overline{FR} ?



- (1) 1
- (3) 36
- (2) 1.2(4) 4
- 3. In the right triangle ABC, $m\angle C = 90$ and altitude \overline{CD} is drawn in hypotenuse \overline{AB} . If AD = 4 and DB = 5, what is AC?
 - (1) $\sqrt{20}$
- (2) 6
- $(3) \sqrt{45}$
- (4) 9

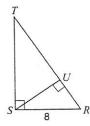
4. In the accompanying diagram, $\triangle RST$ is a right triangle, \overline{SU} is the altitude to hypotenuse \overline{RT} . RT = 16, and RU = 7. What is the length of \overline{ST} ?



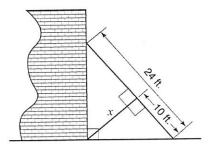
- (1) $3\sqrt{7}$
- (2) $4\sqrt{7}$
- (4) 12
- 5. The altitude drawn to the hypotenuse of a right triangle divides the hypotenuse into segments of lengths 4 and 12. The length of the shorter leg of the right triangle is
 - (1) 8

- (2) $\sqrt{20}$ (3) $\sqrt{48}$ (4) $\sqrt{192}$
- **6.** What is the geometric mean of $\frac{1}{3}x$ and $27x^3$ when $x \ne 0$?
 - (1) $3x^2$
- (2) 9x
- (3) 3x
- $(4) 9x^2$

- the accompanying diagram, $\angle RST$ is a right triangle, \overline{SU} is \pm e altitude to hypotenuse \overline{RT} , 2S = 8, and the ratio of RU to LT is 1 to 3. What is the length of \overline{RT} ?
 - (1) 16
- (3) 24
- (2) 20
- (4) 32



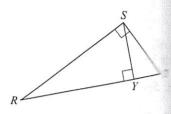
- Show or explain how you arrived at your answer.
- 1. The altitude drawn to the hypotenuse of a right triangle divides the hypotenuse into two segments whose lengths are in the ratio of 1 to 4. If the length of the altitude is 8, find the length of the longer leg of the triangle.



- 9. The accompanying diagram shows a 24-foot ladder leaning against a building. A steel brace extends from the ladder to the point where the building meets the ground. The brace forms a right angle with the ladder. If the steel brace is connected to the ladder at a point that is 10 feet from the foot of the ladder, find to the nearest tenth of a foot the length, x, of the steel brace.
- 10. In right triangle JKL, $\angle K$ is a right angle. Altitude \overline{KH} intersects the hypotenuse at H in such a way that JH exceeds HL by 5. If KH = 6, find the length of the hypotenuse.
- 11. In right triangle ABC, the length of altitude \overline{CD} to hypotenuse \overline{AB} is 12. If the length of the longer segment of the hypotenuse exceeds the length of the shorter segment of the hypotenuse by 7, find the length of the hypotenuse.
- 12. In right triangle ABC, altitude \overline{CD} is drawn to hypotenuse \overline{AB} . If AB is four times as great as AD and AC is 3 more than AD, find the length of altitude \overline{CD} .

- 13. In right triangle ABC, altitude \overline{CD} is drawn to hypotenuse \overline{AB} , AD = 1 and DB is 3 less than CD. Find, in simplest radical form, the perimeter triangle ABC.
- **14.** In the accompanying diagram of right triangle RST, altitude \overline{YS} is drawn to hypotenuse \overline{RT} , RT = 20, TY < YR, and YS = 8.
 - a. If x is used to represent the length of \overline{TY} , write a proportion that could be used to find x. Solve the proportion algebraically for x.

b. Find the length of \overline{ST} in simplest radical form.

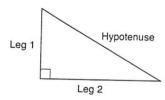


6.2 THE PYTHAGOREAN THEOREM



The Pythagorean Theorem relates the lengths of the three sides of a right triangle:

 $(\text{Leg 1})^2 + (\text{Leg 2})^2 = (\text{Hypotenuse})^2$



The Pythagorean Theorem can be proved using the proportions derived from the Altitude-Hypotenuse Theorem.

A Proof of the Pythagorean Theorem

Although there is more than one way of proving the Pythagorean Theorem. here is a proof that uses proportions in a right triangle.

Theorem: Pythagorean Theorem

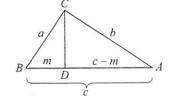
In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse $(a^2 + b^2 = c^2)$.

Proof

 $\triangle ABC$, $\angle C$ is a right angle, c is the length of the hypotenuse, a and b are the lengths of the legs.

e: $a^2 + b^2 = c^2$

The altitude \overline{CD} , and let BD = m, so AD = c - m, as shown in the accompanying Eagram. Then apply Corollary 1:



$$\frac{m}{a} = \frac{a}{c} \qquad \text{so} \qquad a^2 = +mc$$

$$\frac{c-m}{b} = \frac{b}{c} \qquad \text{so} \qquad \frac{b^2 = c^2 - mc}{a^2 + b^2 = c^2} \longrightarrow \text{Pythagorean Theorem}$$

Applying the Pythagorean Theorem

The Pythagorean Theorem can be used to find the length of *any* side of a meht triangle when the lengths of the other two sides are known:

F
$$a = 3$$
 and $b = 5$, then
$$a^{2} + b^{2} = c^{2}$$

$$3^{2} + 5^{2} = c^{2}$$

$$9 + 25 = c^{2}$$

$$c^{2} = 34$$

$$c = \sqrt{34}$$

If
$$a = \sqrt{13}$$
 and $c = 7$, then
$$a^{2} + b^{2} = c^{2}$$

$$(\sqrt{13})^{2} + b^{2} = 7^{2}$$

$$13 + b^{2} = 49$$

$$b^{2} = 49 - 13$$

$$b = \sqrt{36} = 6$$

Example 1

Pay wants to build a square garden in which the distance between opposite corners is at least 18.0 feet. What is the shortest possible side length of the square garden correct to the nearest tenth of a foot?

Solution: Use the Pythagorean Theorem where x represents the length of 1 side of the square garden:

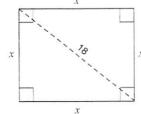
$$x^{2} + x^{2} = 18^{2}$$

$$2x^{2} = 324$$

$$\frac{2x^{2}}{2} = \frac{324}{2}$$

$$x = \sqrt{162}$$

$$x \approx 12.728$$



Round up from 12.728 to 12.8 feet in order for the diagonal length to be at *least* 18.0 feet.

Chapter 6 **RIGHT TRIANGLES AND TRIGONOMETRY**

Katie hikes 5 miles north, 7 miles east, and then 3 miles north again. To the nearest tenth of a mile, how far, in a straight line, is Katie from her starting point?

Solution: The four key points on Katie's trip are labeled A through D the accompanying diagram.

- To determine how far, in a straight line, Katie is from her starting point at A. find the length of \overline{AD} .
- Form a right triangle in which \overline{AD} is the hypotenuse by completing rectangle BCDE as shown in the accompanying diagram.
- Because opposite sides of a rectangle have the same length, ED = BC = 7, and BE = CD = 3. Thus, AE = 5 + 3 = 8.
- Since AD is the hypotenuse in right triangle AED:

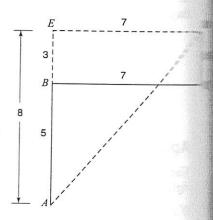
$$(AD)^{2} = (AF)^{2} + (FD)^{2}$$

$$= 8^{2} + 7^{2}$$

$$= 64 + 4$$

$$= 113$$

$$AD = \sqrt{113} \approx 10.63$$



Correct to the nearest tenth of a mile, Katie is 10.6 miles from her starting point.

Pythagorean Triples

A Pythagorean triple is a set of three positive integers $\{x, y, z\}$ that satisfy the relationship $x^2 + y^2 = z^2$. Here are some commonly encountered Pythagorean triples that you should memorize:

Multiplying each member of a Pythagorean triple by the same whole number produces another Pythagorean triple. For example, multiplying each member of {3, 4, 5} by 2 forms a 6-8-10 Pythagorean triple:

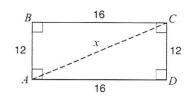
$$\{\underline{3} \times 4, \underline{4} \times 2, \underline{5} \times 2\} = \{6, 8, 10\}$$

Recognizing Pythagorean triples can make problems easier to solve.

Example 3

rectangular yard measures 12 yards by 16 yards. What is the distance from me corner of the yard to the opposite corner?

Solution 1: Draw rectangle ABCD. The lengths of the sides of right triangle *DC are a multiple of the basic 3-4-5 Thagorean triple since $12 = 3 \times 4$ and $65 = 4 \times 4$ so $AC = 5 \times 4 = 20$.



Solution 2: If you did not recognize fat a Pythagorean triple was involved, se the Pythagorean theorem to find the ength of leg \overline{AB} :

$$12^{2} + 16^{2} = (AC)^{2}$$

$$144 + 256 = (AC)^{2}$$

$$(AC)^{2} = 400$$

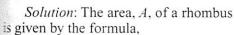
$$AC = \sqrt{400} = 20$$

Area Applications

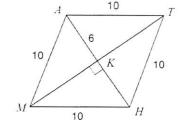
You may need to use the Pythagorean Theorem or a Pythagorean triple to find the length of a segment required in an area formula. A summary of area formulas is included in the section at the back of the book titled, "Some Geometric Relationships Worth Remembering."

Example 4

In the accompanying figure of rhombus *MATH*, diagonals \overline{TM} and \overline{AH} intersect at K, AK = 6, and each side measures 10. Find the area of rhombus MATH.



$$A = \frac{1}{2}d_1d_2$$



where d_1 and d_2 are the lengths of its diagonals.

- The length of the sides of right triangle MKA form a 6-8-10 Pythagorean triple where MK = 8.
- · Since the diagonals of a rhombus bisect each other,

$$d_1 = AH = 6 + 6 = 12$$
 and $d_2 = MT = 8 + 8 = 16$

• Use the area formula $A = \frac{1}{2}d_1d_2$ where $d_1 = 12$ and $d_2 = 16$:

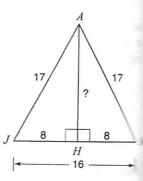
$$A = \frac{1}{2}(12)(16) = 96$$

The area of rhombus MATH is 96 square units.

Example 5

In the accompanying diagram of isosceles triangle JAM, the length of base \overline{JM} is 16 cm. If the perimeter of $\triangle JAM$ is 50 cm, find the area of the triangle.

Solution: The sum of the lengths of the three sides is 50 cm, so the lengths of the two congruent legs add up to 50 - 16 = 34 cm. Therefore, the length of each leg is 17 cm. The altitude drawn to the base bisects the base so that an 8-15-17 right triangle is formed where AH = 15. If you did not recognize that a Pythagorean triple was involved, you could have used the Pythagorean Theorem to find the unknown length.



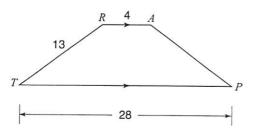
Area of
$$\triangle JAM = \frac{1}{2} \times \text{base} \times \text{height}$$

= $\frac{1}{2} (16 \text{ cm} \times 15 \text{ cm})$
= 120 cm^2

The area of $\triangle JAM$ is 120 cm².

Example 6

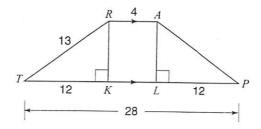
In the accompanying diagram of isosceles trapezoid TRAP, $\overline{RA} \parallel \overline{TP}$, RA = 4 TP = 28, and RT = 13. What is the area of trapezoid TRAP?



Solution: The area, A, of a trapezoid is given by the formula

$$A = \frac{1}{2}h(b_1 + b_2)$$

where b_1 and b_2 represent the lengths of the two bases and h is the altitude.



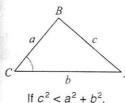
- Draw altitudes \overline{RK} and \overline{AL} thereby forming rectangle RALK. Since KL = RA = 4, TK + LP = 28 4 = 24.
- Because $\triangle TKR \cong \triangle PLA$, $TK = LP = \frac{1}{2}(24) = 12$.
- The lengths of the sides of right triangle TKR form a 5–12–13 Pythagorean triple where altitude RK = 5.
- Use the area formula $A = \frac{1}{2}h(b_1 + b_2)$, where h = 5, $b_1 = 4$, and $b_2 = 28$:

$$A = \frac{1}{2}(5)(4+28)$$
$$= \frac{1}{2}(5)(32)$$
$$= 80$$

The area of trapezoid TRAP is 80 square units.

Pythagorean Inequalities

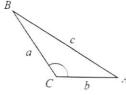
You can tell whether a triangle is acute, right, or obtuse by comparing the squares of the lengths of its sides.



 $C < a^2 + b^2$, $\angle C$ is an acute angle.



If $c^2 = a^2 + b^2$, $\angle C$ is a right angle.



If $c^2 > a^2 + b^2$, $\angle C$ is an obtuse angle.

Given the lengths of a side of a triangle are 4, 5, and 6. Compare the of the largest of the three numbers to the sum of the squares of the numbers:

$$6^2$$
 ? $4^2 + 5^2$

$$6^2$$
 ? $16 + 25$

Because $c^2 < a^2 + b^2$, the triangle is an acute triangle.

Example 7

Classify the triangle whose sides measure $\sqrt{7}$, 3, and 4.

Solution: Compare the square of the largest of the three numbers to sum of the squares of the other two numbers:

$$4^{2}$$
 ? $(\sqrt{7})^{2} + 3^{2}$

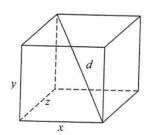
$$16 = 16$$

Because $c^2 = a^2 + b^2$, the triangle is a **right triangle**.

The Pythagorean Theorem In Rectangular Solids

The Pythagorean relationship may be extended to rectangular solids. In the accompanying figure,

$$x^2 + y^2 + z^2 = d^2$$



For example, in a rectangular box whose measurements are 5 inches by 6 inches by 9 inches, the distance, d, from one corner on top of the box to the opposite corner at the bottom of the box is given by

$$5^2 + 6^2 + 9^2 = d^2$$

so
$$25 + 36 + 81 = d^2$$
 and $d = \sqrt{142}$.

Check Your Understanding of Section 6.2

- Choice
- The lengths of the two legs of a right triangle are 2 and $\sqrt{3}$. What is the ent of the hypotenuse?
 - $4\sqrt{3}$
- (2) 10
- (3) $\sqrt{14}$ (4) $\sqrt{7}$
- Large length of the hypotenuse of a right triangle is $\sqrt{15}$ and the length of the leg is 3. What is the length of the other leg?
 - 116
- (2) 9
- (3) $\sqrt{6}$
- (4) $3\sqrt{2}$
- That is the length of a diagonal of a square whose perimeter is 32?
 - $\sqrt{2}$
- (2) $2\sqrt{2}$ (3) $4\sqrt{2}$
- (4) $8\sqrt{2}$
- Thich set of numbers do not represent the lengths of the sides of a right riangle?
 - (1) {9, 40, 41}

(3) $\{\sqrt{12}, 6, 7\}$

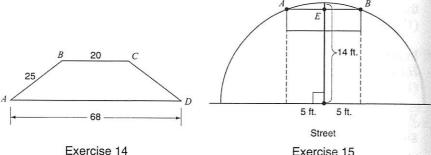
(2) $\{2, 4, 2\sqrt{5}\}$

- (4) $\{1, 1, \sqrt{2}\}$
- . What is the length of a diagonal of a rectangle in which the lengths of two adjacent sides are $\sqrt{13}$ and 6?
 - (1) 7
- (2) 19 (3) $5\sqrt{3}$ (4) $\sqrt{23}$
- Left the lengths of the diagonals of a rhombus are 6 and 8, the perimeter of the rhombus is
 - (1) 14
- (2) 20
- (3) 28
- (4) 40
- 7. In rectangle MATH, AT = 8 and TH = 12. What is the length of diagonal *HA* to the *nearest tenth*?
 - (1) 14.4
- (2) 11.7
- (3) 8.9
- (4) 7.2
- 8. What is the length of the altitude drawn to a side of an equilateral triangle whose perimeter is 60 cm?

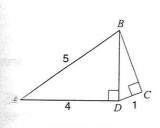
- (1) 10 cm (2) $10\sqrt{2}$ cm (3) $10\sqrt{3}$ cm (4) $10\sqrt{5}$ cm

- 9. A carpenter is building a rectangular deck with dimensions of 16 feet by 30 feet. To ensure that the adjacent sides form 90° angles, what should each diagonal measure?
 - (1) 26 ft
- (2) 30 ft
- (3) 34 ft
- (4) 46 ft
- 10. At 9:00 A.M. a car starts at point A and travels north for 1 hour at an average rate of 60 miles per hour. Without stopping, the car then travels east for 2 hours at an average rate of 45 mile per hour. At 12:00 P.M., what is the best approximation of the distance, in miles, of the car from point A (1) 100(2) 105(3) 108(4) 115
- 11. If the length of each leg of an isosceles triangle is 17 and the base is 16. the length of the altitude to the base is
 - (1) 8
- (2) $8\frac{1}{2}$
- (3) 15
- 12. The lengths of the bases of an isosceles trapezoid are 6 centimeters and 12 centimeters. If the length of each leg is 5 centimeters, what is the area of the trapezoid?

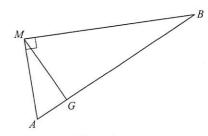
 - (1) 18 cm^2 (2) 36 cm^2
- $(3) 45 \text{ cm}^2$
- $(4) 90 \text{ cm}^2$
- B. Show or explain how you arrived at your answer.
- 13. A baseball diamond is in the shape of a square with a side length of 90 feet. What is the distance from home plate to second base, correct in the nearest tenth of a foot?
- 14. The cross section of an attic is in the shape of an isosceles trapezoid shown in the accompanying figure. If AB = CD = 25 feet, BC = 20 feet and AD = 68 feet, what is the area of the cross section?



- 15. The accompanying diagram shows a semicircular arch over a street that has a radius of 14 feet. A banner is attached to the arch at points A and B in such a way that AE = EB = 5 feet. How many feet above the ground are these points of attachment for the banner? Estimate to the nearest tenth of a foot.
- 16. The perimeter of a rhombus is 100 centimeters and the length of the longer diagonal is 48 centimeters. Find the area of the rhombus.
- The length and width of a rectangle are in the ratio of 3:4. If the length of the diagonal of the rectangle is 60, what are the length and width of the rectangle?
- Two hikers started at the same location. One traveled 2 miles east and then 1 mile north. The other traveled 1 mile west and then 3 miles south. At the end of their hikes, how many miles apart were the two hikers?
- To get from his high school to his home, Jamal travels 5.0 miles east and then 4.0 miles north. When Sheila goes to her home from the same high school, she travels 8.0 miles east and 2.0 miles south. What is the shortest distance, to the nearest tenth of a mile, between Jamal's home and Sheila's home?
- In the accompanying diagram of right triangles ABD and DBC, AB = 5. AD = 4, and CD = 1. Find the length of \overline{BC} , to the nearest tenth.



Exercise 20



Exercise 21

- Town A is 8 miles from town C, town B is 15 miles from town C, and ingle ACB is a right angle. On the straight road that connects towns A and B, a restaurant will be built at the point that is closest to town C. To the nearest tenth of a mile, find
 - The distance from town A to the restaurant
 - The distance from town C to the restaurant